ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
Networked Systems Group (NSG)

HS 2026

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based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 5

1 Revisiting Context-Free Grammars

Consider the context-free languages from last week (cf. Exercise 4.1) on the alphabet $\Sigma = \{0, 1\}$:

a) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

b) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

For each of them, give a context-free grammar in Chomsky Normal Form (CNF) and try finding a grammar with the minimum number of non-terminal symbols.
Bonus: If possible, give a right-linear and a left-linear grammar for the language.

2 Regular, Context-Free or Not?

For the following languages, determine whether they are context free or not. Prove your claims!

a) $L = \{1^k \mid k \text{ prime}\}$

b) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |z|, |x| = |y|\}$

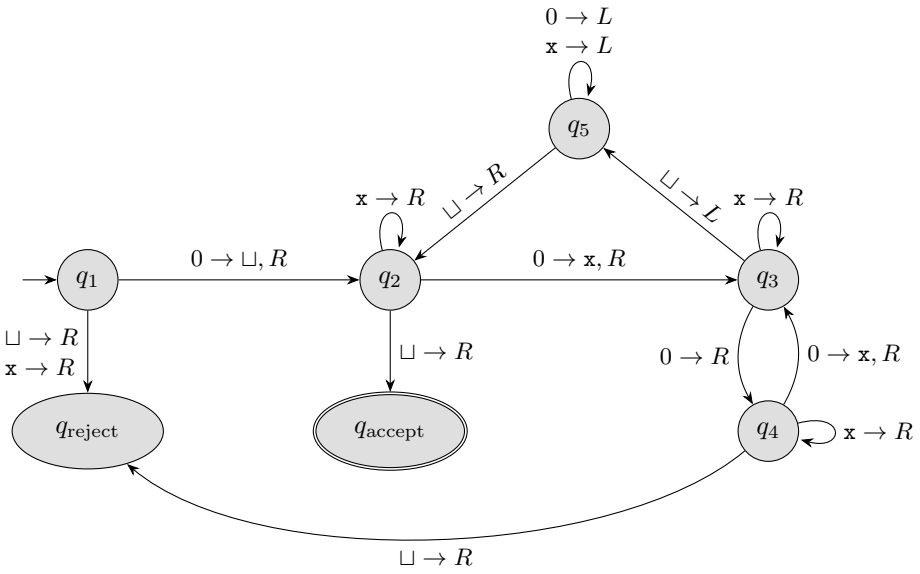
c) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |y|, |x| = |z|\}$

d) $L = \{x \mid x \in \{0, 1\}^*, \text{ and } x \text{ contains an even number of '0's and an even number of '1's}\}$

3 Running a Turing Machine

Consider the following turing machine M_1.

- $Q = \{q_1, q_2, q_3, q_4, q_5, q_accept, q_reject\}$,
- $\Sigma = \{0\}$,
- $\Gamma = \{0, \mathtt{x}, \sqcup\}$
- δ is described by the following state diagram.
- start, accept and reject are $q_1, q_{accept}, q_{reject}$ respectively.



a) For each of the following input strings, give the sequence of configurations that M_1 enters.

(i) 0

(ii) 00

(iii) 000

To denote the state of the machine, write the tape as a string and place the state of the automaton to the left of the cell of the head reading head. Also, you implicitly assume there are infinitely many $\sqcup$ symbols at the right end of each string. For example, the starting state of (i) can be written as $q_1 0$.

b) Describe the language that M_1 recognizes.

4 Tandem-Pumping Lemma [Exam HS21]

Given the alphabet $\Sigma = \{0, 1, \#\}$, consider the language:

$$L = \left\{ a\#b\#c \mid a, b, c \in \{0,1\}^*,\ c = 2a,\ \#_0(b) = \#_0(c) \right\}$$

for unsigned binary numbers a, b, and c. For example, $0\#10\#0 \in L$ and $1\#00\#010 \in L$.
Recall: $\#_0(w)$ denotes the number of occurrences of the symbol $0 \in \Sigma$ in a word $w \in \Sigma^*$.

a) Show that $w = 1^p\#0\#1^p0$ is tandem-pumpable in L.
Hint: Split up $w = uvxyz$ such that $x = \#0\#$.

b) Use the tandem-pumping lemma to show that L is not context-free.
Hint: Choose a string $w = a\#b\#c$ where $1 \notin b$, i.e. $b \in 0^*$.

c) Can we use any string $w = a\#b\#c$ where $b = b_1 1\, b_2$ to apply the tandem-pumping lemma?

5 Java is not regular! [Bonus question]

Prove that the programming language `java` is not regular! More precisely, show that a single statement in `java` cannot be recognized by a regular language.
Hint: Assume that strings in your program do not contain the symbols "{" or "}".