ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
Networked Systems Group (NSG)

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based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 7

1 Üebigstund

The exercise hour (Üebigstund) ist quite demanding to the n students registered to DES. Students will therefore decide to not come to the exercise session randomly and might come back if they feel ready again. The time until a student leaves the room is exponentially distributed with parameter μ (independently of how many students remain in the room). On the other hand, the time until a student enters the room is exponentially distributed with parameter λ.

a) Model the situation as a birth-and-death Markov process where the states denote the number of students in the room.

b) Derive a formula for the probability that exactly i students are in the room, independent of π_0.

Hint:

$$\sum_{i=0}^{n} \binom{n}{i} \cdot x^i = (1+x)^n$$

c) (i) Assume that $n = 5$ students are registered to DES[1], and that $\lambda^{-1} = 9$ weeks and $\mu^{-1} = 3$ weeks. Calculate the probability that the Üebigstund is cancelled because less than 2 students show up.

(ii) How do these probabilities change if the quality of the Üebigstund significantly improves such that $\lambda^{-1} = 4$ weeks and $\mu^{-1} = 3$ weeks?

(iii) For M/M/1 queues, a stationary distribution only exists if ρ is smaller than 1. Why doesn't this hold for this example?

2 Theory of Ice Cream Vending

Apart from their job as PhD students, Jiahui and Jean sell ice cream to further improve their financial situation. In order to serve one customer, each of them needs an amount of time which is exponentially distributed with parameter μ. There is one line of customers in front of their shop, and new customers arrive with a rate λ. Under which conditions is there an equilibrium for this system? And what is the probability that there is no customer in the system (in the steady state)?

[1] in reality it's much more than that ;)

3 Gloriabar

Every day, 540 students, professors and other personnel of ETH go to the Gloriabar for a *delicious* lunch between 11:45 and 13:15. There is only one queue, and the cashier needs on average 9 s to serve a person. Model this situation with an unbounded waiting queue. Assume that the mean interarrival and service times are exponentially distributed and further that we do not model the process in which a customer gets his food.

a) Compute the expected waiting time until a student reaches the cashier and the expected time until she has paid for the food.

b) Compute the expected length of the queue (without the person who is being served).

c) What should be the service time such that the time until a student has paid for her menu is cut in half?